# CHAPTER FOUR

# VARIATION

## Introduction:

This shows the relationship which exists between two variables or quantities.
- For example we may choose to consider the relationship between the number of gallons of petrol used by a travelling car and the distance travelled.
- This relationship or variation will be that, the greater the distance travelled, the greater will be the number of gallons of petrol used.

- Variation truly speaking is the same as proportion.

## Types:

  - Basically there are three types and these are: :

1. Direct variation.

2. Inverse variation.

3. Partial variation.

NB: Associated with each of these is what certain individuals refer to as joint variation

**DIRECT VARIATION :**

-This may also be referred to as direct proportion:

-Two variables or quantities such as x and y are said to vary directly, or are directly proportional if

1. both x and y increases at the same time e.g

| X | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Y | 4 | 8 | 12 | 16 | 20 | 24 |

2. both values of x and y decrease at the same time. i.e X decrease as y decreases. e.g.

| X | 100 | 80 | 60 | 40 | 20 |
|---|-----|----|----|----|----|
| Y | 10 | 8 | 6 | 4 | 2 |

Now consider two variables x and y. If x varies directly as y, then we write

$x \propto y$, i.e x is directly proportional to y.

From $x \propto$ y, in order to remove the proportional sign, we must introduce a constant,

i.e  if $x \propto$ y, $\Longrightarrow$ x = ky, where k = a constant, which may be referred to as the proportionality constant or the constant of proportionality.

Also if M varies directly as R, then M $\propto$ R $\Longrightarrow$ M = K.R, where K is a constant.

Q1. Two variables m and v are such that m varies directly as v. Given the constant as 10, calculate m when v = 20.

Solution

Since m varies directly as v

$\Longrightarrow$ m $\propto$ v, $\Longrightarrow$ m = kv

Where k is a constant.

Since k = 10 $\Longrightarrow$ m = 10v.

When v = 20 $\Longrightarrow$ m = (10)(20) = 200.

Q2. The population, P of a nation is directly proportional to the birth rate, R. If the constant is 20, determine the population in million if R = 5.

Solution.

P varies directly as R

$\Longrightarrow$ p $\propto$ R $\Longrightarrow$ p = KR,

 Where k is a constant. Since k = 20 $\Longrightarrow$ p = 20 R.

When R = 5 $\Longrightarrow$ P = (20) (5) = 100

The population, P = 100 million.

Q3. The velocity v of a car is directly proportional to its mass m squared. Given the constant as 5 and the mass of the car as 60kg, calculate the velocity in m/s.

Solution.

V is directly proportional to m squared $\Longrightarrow$ v $\propto$ m$^2$,

$\Longrightarrow$ v = km$^2$, where k is a constant.

Since k = 5 => v = 5m.²

When m = 60 $\Longrightarrow$ v = 5(60)²,

$\Longrightarrow$ v = 5 (3600) = 1800,

=> V = 18000 m\s.

Q4. The circumference of a circle varies directly as it radius cubed. Determine the circumference in cm, if the radius is 3cm. Take the proportionality constant to be 4.

Solution

Let c = the circumference and r = the radius.

Since the circumference varies directly as the radius cubed

$\Longrightarrow$ C $\propto$ r³ $\Longrightarrow$ c = kr³,

where k = the constant. Since  r = 3cm and k = 4

$\Longrightarrow$ $c = (4)(3)^3 = (4)(27) = 108.$

The circumference = 108cm.

Q5. The density of a material is directly proportional to the square root of its mass. Given the variation constant as 5 and the mass as 25g, determine the density in g cm $^{-3}$.

Let d = density and m = mass.

Since the density is directly proportional to the square root of the mass

$\Longrightarrow$ d $\propto$ $\sqrt{m}$,

$\Longrightarrow$ d = k. $\sqrt{m}$, where k = the constant. Since k = 5 and m =25 $\Longrightarrow$ d = 5.$\sqrt{25}$,  $\Longrightarrow$ d = (5) (5) = 25,

$\Longrightarrow$ d = 25 cm$^{-3}$

Q6. The speed of a vehicle varies directly as it mass. The speed also varies directly

 as its length. Find the speed in km/h, if the car is of mass 120kg and it is 50 m long. Take the variation constant to be 10.

Solution.

Let s = speed, m = mass and *l* = length.

1. The speed varies directly as the mass $\Longrightarrow$ s $\propto$ m…………..(1)

11. Also the speed varies directly as the length $\Longrightarrow s \propto l$.........(2)

From 1 and 2 i.e $s \propto m$ and $s \propto l$

$\Longrightarrow s \propto m.l.$ Removing the proportion sign

$\Longrightarrow s = k.m.l.$

Bu since k = 10, m = 120 and l = 50

$\Longrightarrow s = (10)(120)(50) = 60,000$

$\Longrightarrow s = 60,000$ km/h.

Q7. Three quantities, R, Q and M are such that R varies directly as Q. R also varies directly as M. If the proportionality constant is 20, find R when Q = 1 and m = 7.

Solution

1. R varies directly as Q

$\Longrightarrow R \propto Q$ .

11. Also R varies directly as M.

$\Longrightarrow R \propto M.$

From $R \propto Q$ and $R \propto M$

$\Longrightarrow R \propto Q. M \Longrightarrow R = KQ.M.$

But since Q = 1, M = 7 and k = 20

$\Longrightarrow R = (20)(1)(7) = 140,$

$\Longrightarrow R = 140.$

Q8. The velocity of a car varies directly as its mass squared. It also varies directly as the length of the car. Given that the car has a length of 10m and a mass of 30kg, calculate its velocity in km/h. Take the proportionality constant as 200.

Solution

Let v = velocity, m = mass and l = length.

1. Since the velocity varies directly as the mass squared

$\Longrightarrow v \propto m^2.$

2. Also since the velocity varies directly as the length

$\Longrightarrow$ V $\propto$ l.

From 1. V $\propto$ m² and from 2. V $\propto$ l, $\Longrightarrow$ v $\propto m^2 l \Longrightarrow v = km^2 l$.

But m = 30, k = 200 and l = 10 .

From v = km²l $\Longrightarrow$ v = (200) (30)²(10),

$\Longrightarrow$ v = (200) (900) (10),

$\Longrightarrow$ v = 18 000 00 km/h.

Q9. The quantities Q, L and Z are such that L varies directly as Q cubed as well as Z. Given that L = 160 when Q = 2 and Z = 5, calculate the variation constant.

<div align="center">Solution</div>

1. L varies directly as Q cubed $\Longrightarrow$ L $\propto$ Q³.

11. L varies directly as Z $\Longrightarrow$ L $\propto$ Z .

Now from L $\propto$ Q³ and L $\propto$ Z

$\Longrightarrow$ L $\propto$ Q³. Z, $\Longrightarrow$ L = K.Q³. Z.

But since L= 160, when Q = 2 and Z = 5, $\Longrightarrow$ L= k $(2)^3(5)$,

=> 160 = K(8)(5) $\Longrightarrow$ 160 = 40K,

$\Longrightarrow$ K = 160/40 = 4.

The constant = 4.

10. The distance d covered by a vehicle varies directly as its length, l cubed as well as the square root of its mass m. Given that when m = 9kg, l = 2m and d = 120km,

1. determine the value of the constant.

11. deduce an expression for d in terms of m, l and k, where k is the constant

111. determine the distance travelled by the car in kilometers, when it is 40m long and has a mass of 60kg.

<div align="center">Solution</div>

1.Since d varies directly as l cubed $\Longrightarrow$ d $\propto l^3$

11. Since d also varies directly as the square root of m

$\Longrightarrow$ d $\propto$ √m.

Now  since d $\propto$ $l^3$ and d $\propto$ √m,

$\Longrightarrow$ d $\propto$ $l^3$.√m => d = k.$l^3$. √m..

But  d = 120 when l = 2 and m = 9, and since d = k $l^3$√m,

$\Longrightarrow$120 = k(2)³(√9) = k(8)(3) = 24k,

$\Longrightarrow$120 = 24k $\Longrightarrow$ k = $^{120}/_{24}$ = 5.

∴ K = 5.

11. We have already noted that d = kl³√m.

To deduce an expression for d in terms of k, l and , m, we only substitute the value of the constant (i.e. 5) into the just written expression.

From d = kl³ √m $\Longrightarrow$d = 5l³√m.

111. When l = 40m and m = 60kg,

$\Longrightarrow$ d = 5 l³ √m $\Longrightarrow$d = 5(40)³ √60,

$\Longrightarrow$ d = 5 (64000) (7.7),

$\Longrightarrow$ d = 2,464 000.